Decision Support Systems for Portfolio Optimization

María A. Osorio Lama, Abraham Sánchez López

School of Computer Sciences, Universidad Autónoma de Puebla,
Ciudad Universitaria, Puebla, Pue. 72560, México

{aosorio, asanchez}@cs.buap.mx
http://www.cs.buap.mx/~aosorio

Abstract. We present the elements for Decision Support Systems for Portfolio Optimization, including the financial investment elements for a knowledge database and the elements for a Portfolio Optimization Model. We take into account the tax impact and the investor's scenario role into the models, and modify the classic Markowitz model, augmenting constraints that consider them and proposing stochastic linear and stochastic quadratic models. The efficient frontier is calculated using the augmented model. Finally we present the elements for an integrated knowledge and preference systems with the Optimization model.

1 Introduction

We are in the midst of a revolution in investment management. An unprecedented globalization of financial markets, advances in the electronic transmission of data, the accessibility of inexpensive yet extremely powerful computer hardware and software, and the migration during the past decade of so-called quants and computer wizards to Wall Street have all contributed to this revolution according to Aiken [1]. The body of finance theory and empirical evidence related to rational investment decision making has become so large that any future advances are expected to be incremental. Thus, the coming decades will likely bring a consolidation and accelerated application of this knowledge (see Doyle [2] and Min [3]).

The computerization of investment decision-making activities requires software systems that integrate mathematical models, a source of data, and a user interface. Such systems are generally referred to as decision support systems, or DSSs. That form of DSS whose database includes relevant theory, facts, and human knowledge and expertise is called knowledge-based system, also referred to as an intelligent system or expert system (ES). Specific techniques such systems employ to acieve their goals are drawn form the field of study known as artificial intelligence (AI) (see Pomerol [7]).

The renewed interest in applying knowledge-based systems to business decisions can be attributed mainly to the plummeting costs of hardware and software. Financial applications are now viewed as ideal proving grounds for new AI concepts and products, because in the realm of finance, significant, rapid, and easily measurable economic benefits are often possible.

© A. Gelbukh, R. Monroy. (Eds.)
Advances in Artificial Intelligence Theory
Research on Computing Science 16, 2005, pp. 13-22

Initially knowledge-based systems were viewed as tools to enable nonexperts to make decisions as effectively as would one or more experts in a particular field, or domain.

How would and intelligent system function in the portfolio investment domain? Consider the problem of how to allocate a large sum of money among stocks, bonds, real estate, and precious metals. Early ESs used for this purpose would probably have included in their data bases the knowledge of several human experts in each of these investment areas. Such a database is called a knowledge base. Today the knowledge base would be likely be built, at least in part, by a machine learning-based subsystems utilizing rule induction, genetic algorithms, or some other paradigm of learning automation. A money manager using such a system could, in theory, manage a portfolio including several asset classes more effectively and at less cost that could any of the individual domain experts. The most obvious advantages of integrating some form of computer intelligence into the portfolio decision-making process over continually consulting with a team of experts include permanence, usually a much lower cost, and a greater consistency of results.

We present the financial investment elements for a knowledge database in section 2, including definitions taxes impact and investor's scenarios. In part 3 we describe the elements for a Portfolio Optimization, taking into account the tax impact and the investor's scenario into the constraints in the model, modifying the classic Markowitz model, proposing stochastic linear and stochastic quadratic models. We also present the efficient frontier calculated with the augmented model. In section 4, we present the elements for an integrated knowledge and preference systems with the Optimization Model, and the conclusion and an example in section 5.

2 Financial Investment Elements for a Knowledge Database

We present the definitions and elements that should be present in the knowledge database, considering taxes ant the investor's scenario. They were first introduced in the models by Osorio et al [5, 6].

2.1 Definitions

Portfolio: A set of assets available for the investor.
Assets: The assets considered are Equities, Bonds, Cash and Properties in the United Kingdom or abroad, available for the constitution of a portfolio distribution.
Returns (Dividends or Income): Percentage of returns in the form of dividends for equities, bonds or properties and income for cash assets.
Capital Gains: Percentage of growth in the capital value of the assets included in the portfolio.
Net Redemption Value: Total amount of money received at the end of the horizon, when a wrapper is encashed, and taxes are paid.
Wrapper: A wrapper is a set of assets (Equities, Bond, Cash and Properties) with a specific set of rules for taxation on regular basis and in different investor scenarios.

2.2 Wrappers

Generally, wrappers can be divided on three general classes:

Offshore Bond: An offshore bond is a foreign non-income producing asset for a national investor. Life insurance companies in offshore centres usually offer offshore life insurance bonds. The major difference between national and Offshore insurance bonds is the location of the insurance company managing the funds and the tax treatment of the bond. The main advantage of an offshore bond is that the investment grows more or less free of tax, although the funds may suffer withholding taxes on their income. This is often knows as 'gross roll-up' and, over the long term, the compounding effect of this can make a substantial difference to the eventual overall return, despite the higher tax on final encashment.

Onshore Bond: Money is invested in a wide range of assets. These may include equities, bonds, cash and commercial property, the value of which will fluctuate. Onshore bonds have several important features as an investment medium. It is possible to switch from one asset to another without any personal capital gains tax charge arising. Usually tax on their income and capital gains is payable at a reduced rate equal to basic rate tax.

There may be capital gains tax advantages as the price of units in the funds allows for the future liability to tax on capital gains.

Unit Trust: Unit trusts offer an opportunity for capital growth with a much greater spread of risk than that usually associated with direct equity investment. A unit trust pools the resources of its individual investors and fund managers, and invests the total within a defined range of investment markets. The fund is divided into equal proportions called units, the price of which is normally quoted on a daily basis. Unit trusts normally pay dividends to investors twice a year.

The value of a unit trust is determined by the price of these units, which is calculated daily by reference to the value of the securities and other assets held in the fund. The value of those units can go down as well as up and investors may not get back their original investment.

There are general and specialized trusts and the permitted areas of investment will be set out in the trust deed. Funds can be geared to produce an income and/or capital growth. The type of units will determine how the dividends are received. With income units, they are paid out direct to the unit holder. With accumulation units the dividends are reinvested in the unit trust and contribute to the growth of the capital value of the funds held and thus increase the unit price. a specific set of rules for taxation on regular basis and in different investor scenarios.

2.3 Taxes

Taxes are payable in different ways for different wrappers and assets. Tax is evaluated in view of specific situations and the wrapper utilized. The main taxes applicable in this paper are:

Income tax: This tax is annually paid in the Unit Trust wrapper for the returns received that year.

Capital Gains Tax: Profits realized on the disposal of certain types of assets and wrappers are subject to capital gains tax. These include direct holdings of shares and property, as well as unit trust. However, in some countries, there is some mitigation of this tax in the form of taper relief. Taper relief is a gradual reduction of the amount of tax payable, dependent on the length of time an asset has been held.

Tax within bond: Tax charged within a fund. This is the case for the wrapper Onshore Bond where a tax charge arises annually within the fund, which provides the investor with a 'credit' for the basic rate of income tax.

Tax on Gross Returns: This tax is paid on the encashment of Onshore or Offshore Bonds wrappers. On Offshore bond, no tax arises on income or gains until exit, but on encashment, all income and gains are taxed at income tax.

Table 1. Type of Taxes

Wrappers	Annual	When Encashed	Withdrawals from
None	Income Tax	Capital Gains Tax	Income (NO) Capital Gains
Offshore Bond	No	Tax Offshore End	Allowed (NO) Returns (Tax Onshore)
Onshore Bond	Tax Onshore	Tax Onshore End	Allowed (NO) Returns (Tax Offshore)
Unit Trust	Income Tax		Income (NO) Capital Gains

2.4 Investor's Scenario

Investor events are the combination of different real life situations that require specific taxation rules and can affect the net redemption value of a wrapper. These events are:

Withdrawals: Withdrawals can be made anytime but are grouped in an annual basis in our model, for tax purposes. Withdrawals are split between the different wrappers and assets. In Unit Trusts, returns coming from dividends or income are subject to immediate tax and may be withdrawn without any further tax, but the capital gains are subject to capital gains tax.

Gift: A gift will not cause any taxation in Onshore and Offshore bonds but will incur in income tax and capital gains tax in Unit Trust.

Death and Inheritance: In the case of death with a testament, there is an inheritance tax on the total value of the wrapper in any wrapper. No Capital Gains tax is paid in wrapper Unit Trust, in the year of death.

Emigration: It only makes a difference in Offshore Bonds, where relief is given on a pro-rata basis for any time spent as non-national resident for tax purposes during the duration the bond is held as an investment, and there is a potential to 'empty' tax-free if the bond is encashed outside the nation after one year's non residence. tax.

Table 2. Taxes according to the Investor's Scenario

Scenario	None	Onshore Bond	Offshore Bond	Unit Trust
Gift	Tax on Dividends, Income, Capital Gains	No Tax	No Tax	Tax on Dividends, Income, Capital Gains
Death+ Inheritance	Inheritance Tax. No tax on Capital Gains	Tax on Gross Bond Returns + Inheritance Tax	Tax on Gross Bond Returns + Inheritance Tax	Inheritance Tax. No tax on Capital Gains
Withdrawals	Tax on capital gains for the withdrawal	A percentage deferred	A percentage deferred	Tax on capital gains for the withdrawal
Emigration	No Exception	No tax on Gross Bond Returns	No tax on Gross Bond Returns	No Exception

3 Elements for a Portfolio Optimization Model

From a global point of view, we should consider the elements shown in Figure 1 in a portfolio optimization model.

3.1 Constraints

Initial Allocation. At the beginning, the initial investment is distributed among instruments within each wrappers.

Cash Balance Equations. Subsequent transactions do not alter the wealth within the period in question. Therefore the following constraints specify to balance the portfolio for each wrapper at each node. These constraints basically balance the cash by reallocating money among assets within each wrapper. In this way, sales fund the purchase of other assets in the same wrapper.

Wealth Balance. The wealth for each wrapper is described by the wealth of the previous year plus the increased value of the assets in that wrapper after paying the corresponding annual taxes. The transaction also allows selling or buying assets within the same wrappers, with the corresponding transaction costs.

Cumulative Taxes. Cumulative tax is paid at the end of the investment horizon when the investment is encashed. At the beginning of the investment plant, the cumulative tax for each wrapper is assumed to be zero.

Cumulative Returns. Returns include the income or dividend after income tax and the capital gain tax are deducted.

Diversification Constraints. Any diversification restriction imposed by the investor or the bank's advice can be expressed by the percentage upper bounds for each asset within each wrapper.

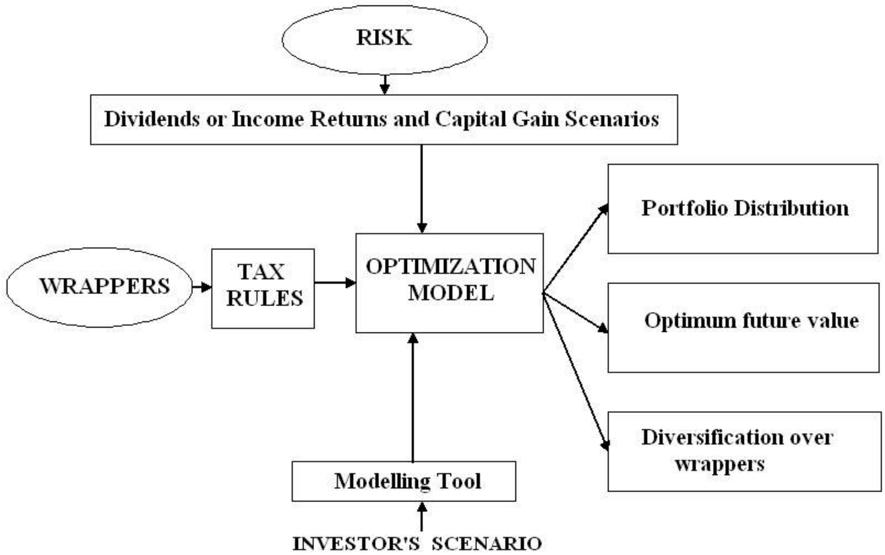

Fig. 1. Elements for a Portfolio Optimization Model

3.2 Uncertainty Representation and Scenario Trees

These models require a coherent representation of uncertainty. This is expressed in terms of mutivariate continuous distributions. Hence, a decision model is generated with internal sampling or a discrete approximation of the underlying continuous distribution. The random variables are the uncertain return values of each asset on an investment. The discretization of the random values and the probability space leads to a framework in which a random variable takes finitely many values. At each time period, new scenarios branch from the old, creating a scenario tree. Scenario trees can be generated based on different probabilistic approaches as simulation or optimization as presented in Gulpinar [4].

3.3 Multistage Optimization Models

Using the constraints described in 3.1 and the uncertainty in the model represented by scenario trees, we can define different optimization models. These models will all be multistage because they use the wealth generated in the previous period in order to

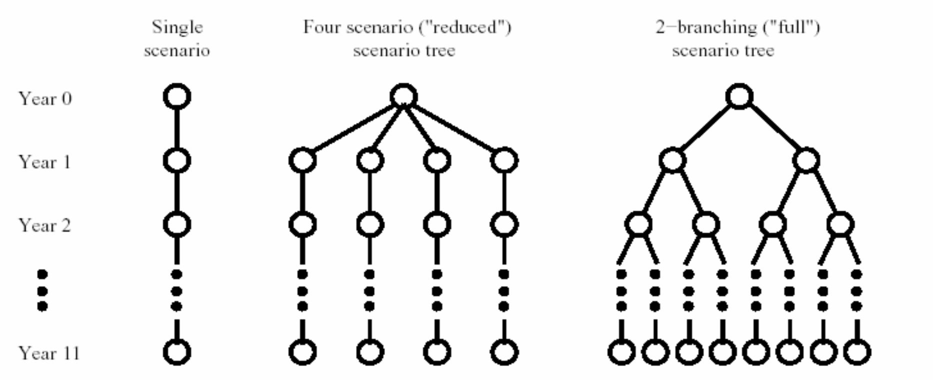

Fig. 2. Scenario Trees for Multiperiod Optimization

represent the constraint in the next period. If take into account the uncertainty of the assets return, based in the history of each asset and represent it in scenario trees models will be stochastic, otherwise they will be deterministic. According to the objective function, models generated under these concepts can be classified as:

A Stochastic Multistage Linear Programming Model. The expected post-tax wealth for each wrapper is calculated as net redemption value at the end of the investment period. In all time periods, taxes are accumulated and deferred, which must be paid on encashment of wrappers. The net redemption value for wrappers is then computed by deducting the accumulated taxes from the final portfolio. The total expected post-tax wealth at the last time period of the investment horizon is the sum of net redemption values of all wrappers and is the *mean*. Stochastic linear programming models which does not incorporate the quadratic variance term, computes the maximum expected return as the mean. This is the objective function for this model.

A Stochastic Multistage Quadratic Programming Model. Gathering together the variance terms of all wrappers, at all time periods will give the total variance in the model. Stochastic quadratic programming models with only wealth balance constraints were originally defined by Markowitz. This model can be enhanced considering all the constraints mentioned in the previous and risk in conjuction with the mean variance which is the objective function for this model (SQP).

Both models can be mixed if we allow the user to withdraw money from the investment's initial amount even instead of forcing the withdrawals only from the returns generated in the investment.

3.4 The Efficient Frontier

The Efficient Frontier is defined as the set of efficient points. Enlarging the universe of assets within wrappers from which the portfolio selection is made never results in a lower efficient frontier, since new securities can always be included at a level of zero. By including in a portfolio new assets whose returns are not highly positively correlated with those of other assets or wrappers, investors may develop significantly improved risk-return combinations. This explains the current trend toward global investing and the inclusion o real estate an other nontratidional assets in the portfolios of major institutional investors.

Which portfolio is chosen from among those on the efficient frontier will depend on the investor's utility function, which represents preferences with respect to risk and return. Different investors facing and identical efficient frontier are likely to choose efficient portfolios having at least somewhat differing levels of risk and expected return.

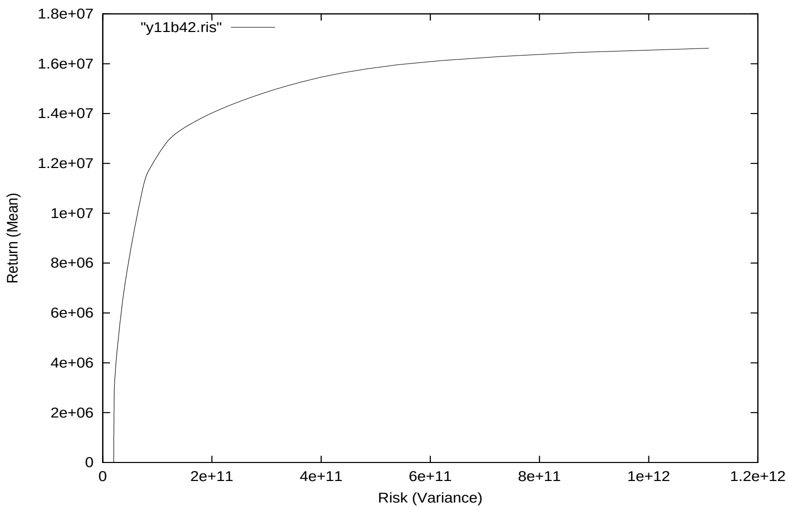

Fig. 3. Efficient Frontier: Mean vs Variance time.

4 Integrating Knowledge and Portfolio Optimization Models

Finally the elements in the portfolio optimization models and the knowledge databases can be integrated in the scheme showed in Figure 4. The scheme presented is similar to the scheme first introduced by Trippy and Lee [8], but instead of the prioritized variables he suggests, we generated additional constraints with enriched elements to manage a wide range of withdrawals in different wrappers for the portfolio elements considered, and took into account the investors' scenario first introduced by Osorio et al [5].

5 Conclusion

We will consider a real life case with the data in Table 3.

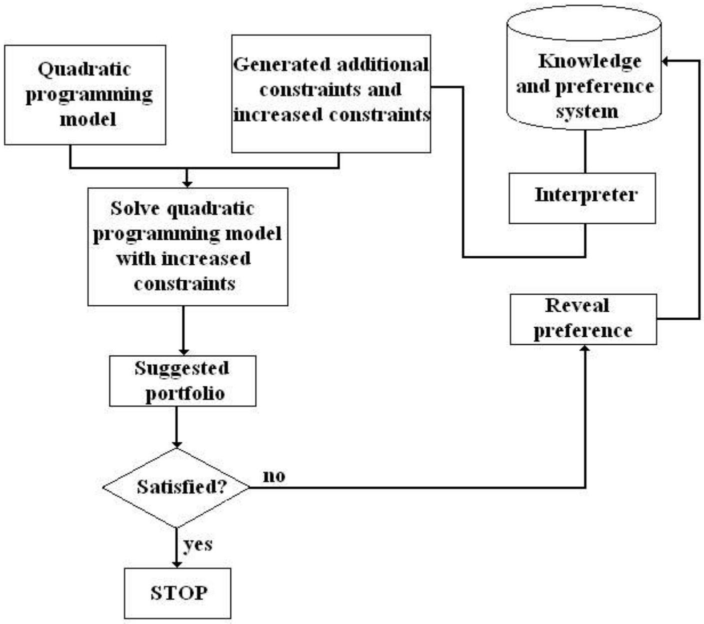

Fig. 4. Integrating Knowledge and Portfolio Optimization Models.

Table 3. Data for the example

YEAR	WITHDRAWAL	CGT	TAXES	%
1	£ 500,000	40%	Tax on Offshore Bond (end)	40%
2	£ 500,000	40%	Tax on Onshore Bond (annual)	22%
3	£ 500,000	40%	Tax on Onshore Bond (end)	18%
4	£ 500,000	38%	Income Tax (Cash)	40%
5	£ 500,000	36%	Income Tax (GBP Equities)	25%
6	£ 500,000	34%	*COSTS*	
7	£ 500,000	32%	Transaction Cost (all wrappers)	1%
8	£ 500,000	30%	Annual Cost (all wrappers)	1.15%
9	£ 500,000	28%	Initial Cost (all wrappers)	0%
10	£ 500,000	26%	*BOUNDS*	
11		24%	On all assets	43%

In this example, an investor sold a factory and would like to invest £8,000,000 and to live from this money the next eleven years. He would like to get an annual withdrawal of £500,000. How would he have to invest his money in order to maximize the money he will receive back after eleven years? To solve this question, we tested our approach using several structure for the trees. Table 4 shows the Net Redemption Value obtained with each and the number of constraints and variables that used each tree.

Table 4. Results for the example

MODEL	LP	MIP	LP
TREE	Reduced	Reduced	Full
NODES	44	44	4094
Constraints	1370	1610	112,574
Variables	3250	3730	300,654
Binaries	0	120	0
NRVW	11,324,784.76	12,201,189.91	11,459,678.32
CPU Secs	18.73	3925.84	63,424.32

As a conclusion we can say that optimal investments strategy generally entails a diversification over different assets. The more exact model is integer (MIP) and requires the solution of an LP in every node of the searching tree. The linear model only solves one LP. Our experiments show that the MIP model generally yields better returns and is therefore preferable despite the higher computational cost.

Acknowledgements. Research supported by SEP-BUAP-VIEP Project No. III 45-04/ING/G.

References

1. Aiken, M., Liu-Sheng, O., Vogel D.: Integrating Expert Systems With Gorup Decision Support Systems. ACM Trnasactions on Information Systems, Vol. 9, No. 1, January 1991, USA (1991) 75-95.
2. Doyle, J., Dean, T. et al: Strategic directions on Artificial Intelligence. ACM Computing surveys, Vol. 28 No. 4. USA (1996) 653-670.
3. Min, H., Eom, S.: An Integrated Decision Support System for Global Logistics, International Journal of Phisical Distribution & Logistic Management, Vol. 24, No. 1 MCB University Press USA (1994) 29-39.
4. Gülpinar, N., Rustem, B., Settergren, R.: Optimization and Simulation Approaches to Scenario Tree Generation. Journal of Economics Dynamics and Control, Vol. 28, Issue 7 (2004) 1291-1315.
5. Osorio, M.A., Settergren, R., Rustem, B., Gulpinar, N.: Post Tax Optimal Investments, Financial Engineering, e-Commerce ans Supply Chain. Kluwer Academic Publishers (2002) 153-173.
6. Osorio, M.A., Gulpinar, Settergren, R., Rustem, B.: Post-Tax Optimization with Stochastic Programming. European Journal of Operational Research, Vol. 157 (2004) 152-168.
7. Pomerol, J.C.: Tutorial on Artificial Intelligence and Human Decision Making, Euro XIV Conference on Operational Research, Jerusalem, Israel (1995).
8. Trippy, R., Lee, J.: Artificial Intelligence in Finance and Investing: state-of-the-art technologies for securities selection and portfolio management. IRWIN Professional Publishing, USA (1996).